percentiles of capability. The multiphase roll programs appear to yield lateral ranges within several percent of the sophisticated steepest-descent solution, yet require only about $\frac{1}{50}$ the computation time and effort.

References

[1] Bryson, A. E., Mikami, K., and Battle, C. T., "Optimum lateral turns for a re-entry glider," Aerospace Eng. **21**, 18–23 (March 1962).

[2] Wagner, W. E., "Optimum trajectory program, UB-057 Part I—Program formulation," Martin Co., Space Systems Div., Rept. ER 12940 (April 1963).

[3] Wagner, W. E. and Jazwinski, A. H., "Three-dimensional re-entry optimization with inequality constraints," AIAA Preprint 63-419 (August 1963).

[4] Jackson, W. S., "An improved method for determining the lateral range of a gliding entry vehicle," J. Aerospace Sci. **28**, 910–911 (1961).

[5] Slye, R. D., "An analytical method for studying the lateral motion of atmospheric re-entry vehicles," NASA TN D-325 (September 1960).

SEPT.-OCT. 1965 J. SPACECRAFT VOL. 2, NO. 5

An Introduction to Gyro Optical Pickoffs

J. Richard Vyce*
Perkin-Elmer Corporation, Norwalk, Conn.

An optical pickoff provides a uniquely satisfactory solution to the problem of sensing the attitude of aerostatic, electrostatic, and cryomagnetic field-suspended spherical-rotor gyros that are invariably operated in the space-fixed mode. The servos in gimbaled gyros are controlled by the optical pickoff to maintain the pickoff aligned with the rotor, usually parallel to the rotor spin axis. Autocollimation permits true angle sensing uninfluenced by rotor translation, but an autocollimating pickoff requires a flat mirror on the rotor and normal to its spin axis. Alternatively, in cases where the rotor surface must be perfectly spherical, an autoreflector viewing a variable reflectivity pattern at a pole is used to sense polar translation, which is proportional to the angular misalignment of a center-fixed rotor. Field-suspended, strap-down gyros, that invariably employ spherical rotors, utilize an orthogonal set of autoreflecting pickoffs to sense the transit of a rotor surface pattern and a computer to determine the orientation of the spin axis from the pickoff signals. Autocollimator accuracy is shown to fall in the $\frac{1}{10}$ to 1 arc-sec region, whereas axial autoreflector accuracy is about 1 μin.; the strap-down readout system is capable of measuring with 10 to 100 arc-sec accuracy. Autocollimator and axial autoreflector accuracies are limited principally by the pickoffs themselves, whereas strap-down readout accuracy is restricted by the precision with which the sensing pattern can be applied to the rotor.

Introduction

ADVANCED gyro developments of recent years include a number of exotically levitated forms,[1] most of which introduce new and rather severe constraints to the problem of measuring rotor orientation. Briefly, the constraints arise from 1) the rotor isolation necessitated by the suspension system, 2) the inherent two-axis character of space-fixed gyros, 3) their exceedingly low-suspension torques, which made them highly susceptible to pickoff torques, and 4) the purely spherical rotors required in some configurations.

The most common gyro configuration involves a readout system usually consisting of angular position indicators on gimbal axes which is controlled to track the rotor by error signals from a null-sensing pickoff. The pickoff serves to indicate extremely minute rotor angular deviation from a fixed (null) relation to the pickoff, and, in view of the forementioned constraints, must 1) be extremely accurate, 2) indicate in two axes, 3) be capable of relatively remote operation, and 4) exert negligible torque and force at the rotor. An optical null-sensing pickoff, because of its accuracy, its remote operating capability, and its low reaction force, presently meets these requirements best and is, therefore, widely employed for exotic gyros. Two types are the autocollimator and the autoreflector. The former is generally preferable, but it requires that the rotor have a flat mirror surface normal to its spin axis. This feature can be accommodated by gas and cryomagnetic gyros, but the purely spherical rotor of the electrostatic gyro forces the use of an autoreflecting pickoff.

The relatively new strap-down gyro involves a space-fixed rotor whose attitude is measured by a set of direct-readout pickoffs that is at all times randomly related to it. Whereas the null-sensing pickoff indicates only small rotor misalignment, the direct-readout pickoff system must actually measure rotor attitude throughout its possible range of motion. The pickoffs themselves are autoreflectors that sense the transit of a line pattern on the rotor with extremely precise time resolution. A digital computer determines rotor orientation from the pickoff inputs. This strap-down, direct-readout system poses numerous problems for the pickoff and rotor pattern, as would be expected inasmuch as they essentially perform the combined function of the pickoff and gimbal-angle transducers on gimbaled gyros. (The strap-down gyro may be employed in place of a conventional gyro in any application and is not necessarily confined to a strap-down guidance system. Conversely, strap-down guidance systems may utilize conventional gyros. The strap-down designation simply indicates that certain functions formerly performed mechanically, such as rotor orientation measurement and accelerometer output resolution, are now done analytically.)

Presented as Preprint 63-314 at the AIAA Guidance and Control Conference, Massachusetts Institute of Technology, Cambridge, Mass., August 12–14, 1963; revision received April 15, 1965.

* Engineering Manager, Metrology; now Manager, Special Projects, Itek Corporation, Lexington, Mass.

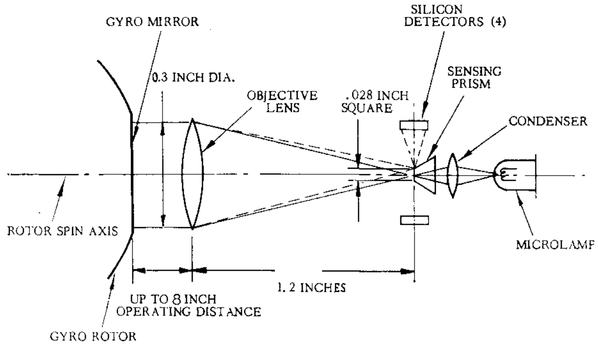

Fig. 1 Autocollimator (OPTAG I) schematic diagram.

Null-Sensing Pickoffs

The Autocollimating Pickoff

Operating principles

Figure 1 represents schematically the Perkin-Elmer OPTAG I optical pickoff. Light from a special microlamp is condensed to fill the clear nose of the truncated pyramidal sensing prism. Light passing through the nose is collimated by the objective lens, transmitted to the rotor mirror, and reflected back to the objective that re-images, or autocollimates, the illuminated nose in its own plane at unity magnification. If the rotor mirror is not precisely normal to the autocollimator axis, the nose image (Fig. 2) is displaced laterally, and light is reflected from a prism side to a detector. The two detectors in each channel provide signals of opposite polarity to indicate the direction of mirror deviation. The final autocollimator outputs (Fig. 3) indicate the direction and magnitude of rotor deviation about two orthogonal axes normal to its spin axis. The autocollimator's output is determined only by mirror angular deviation and is not influenced by mirror translation along or transverse to the line of sight. Translation of the mirror in its own plane clearly does not influence the condition of autocollimation, since one part of its surface appears the same as another; translation normal to the line of sight tends to change the external optical gain by a mechanism called vignetting. Vignetting, however, does not produce a change in angular indication, and at the short, optical, pickoff operating distances, even its effect on gain is negligible.

Performance evaluation

The two characteristics of most immediate interest in a null-sensing pickoff are angular resolution and null stability. Angular resolution is usually established as that angle at which the autocollimator signal-to-noise ratio is equal to unity within a specified measuring bandwidth that we will take to be 1 cps. Pickoff angular resolution determines the short-term uncertainty in indicated rotor attitude. Null stability is a measure of the drift with time of the null point and determines the long-term uncertainty in indicated gyro attitude.

The performance analysis (Fig. 1) is as follows. The radiometric sensitivity

$$P_\theta = NK_L\omega_0A_\theta K_TK_\lambda K_V \simeq 10^{-8} \text{ w/arc-sec}$$

where

P_θ = the radiometric sensitivity in w/arc-sec within the 0.6- to 1.0-μ spectral region
N = radiance of a W filament at 2150°K corresponding to a life of approximately 50,000 hr for 1.5 mil wire[2] = 80 w/in.^{2} sr
K_L = filament packing factor = 0.3
ω_0 = solid angle of objective lens at sensing prism nose = 0.05 sr
A_θ = swept area of autocollimated nose image per arc-sec of mirror deviation = 3×10^{-7} in.^{2}/arc-sec
K_T = optical transmission efficiency (lamp-to-detector) = 0.28
K_λ = spectral utilization factor (lamp-detector) = 0.1
K_v = over-all vignetting factor = 1 for small field angle and short operating distance.

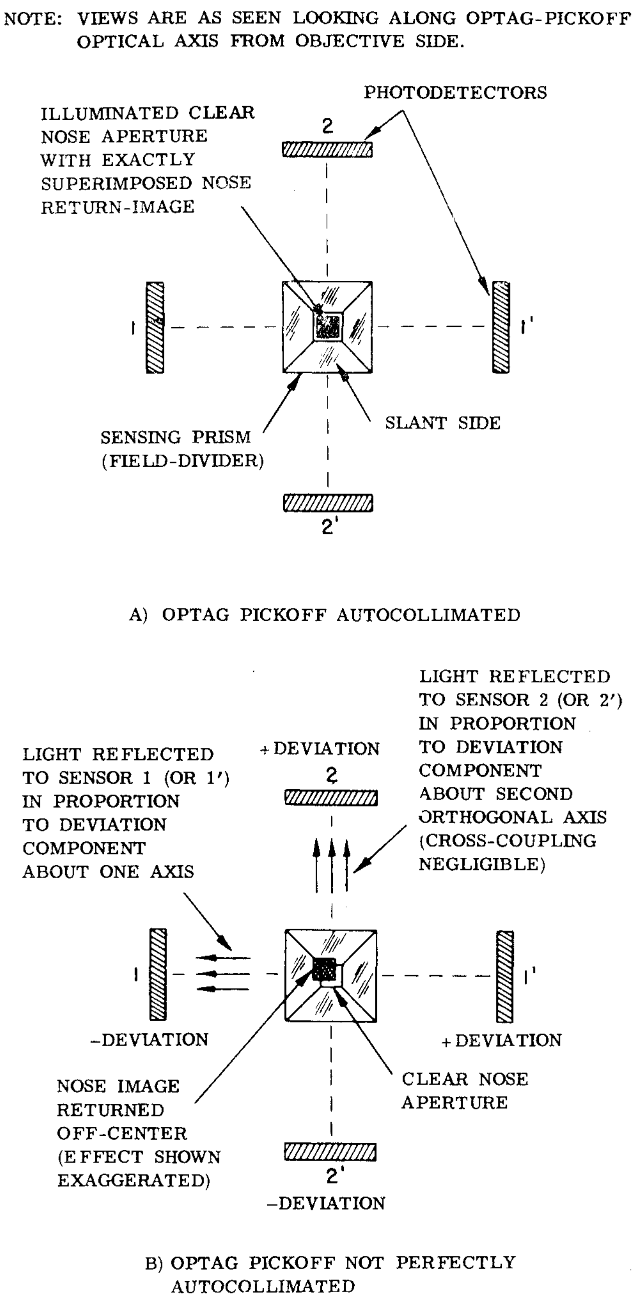

Fig. 2 Autocollimator signal generation diagram.

The one unavoidable source of optical noise in an autocollimator is the power in nose image "spillover" due to diffraction. For a 0.3-in. aperture, at 0.8 μ, the diffraction pattern is approximately 26 arc-sec in diameter. Therefore, P_D = power in diffraction pattern $\cong 26 \times 10^{-8}$ w. Assuming that the main noise sources are fluctuations in the detector photocurrent produced by this diffracted power, we find that the noise power is given by

$$P_N = P_D\left(\frac{2\Delta f}{i_D}\right)^{1/2} = \left(\frac{2P_D{}^2\Delta f}{K_PQP_D}\right)^{1/2} = \left(\frac{2P_D\Delta f}{K_PQ}\right)^{1/2} = 5 \times 10^{-13}\text{w}$$

where

P_N = the noise power in watts
i_D = the diffraction pattern photocurrent in electrons per second
K_P = photon conversion factor = 4×10^{18} photons/sec-w at 0.75 μ
Q = 0.5 photoelectrons/photon for a silicon detector[3]
Δf = information bandwidth = 1 cps.

This is somewhat below the noise level of the best available solid-state detectors. As better detectors are developed, shot noise will become the primary resolution-limiting factor. Assuming a realistic detector noise-equivalent-power (NEP) of 10^{-11} w for a 1-cps bandwidth, we find that the autocollimator noise-equivalent-angle (NEA) has a limiting value of NEA = NEP/P_θ = $10^{-11}/10^{-8} = 10^{-3}$ arc-sec.

The preceding analysis shows that the most significant noise source is the detector NEP assumed to be 10^{-11} w, and that OPTAG I, with a sensitivity of 10^{-8} w/arc-sec, should be capable of resolving a mirror deviation of about 10^{-3} arc-sec. The best observed resolution (OPTAG 1C) has approached 10^{-2} arc-sec. The disparity is due, primarily, to internally scattered light, imperfections in the sensing prism, air shimmer effects, and electronic noise sources, none of which is fundamental, and all of which can be reduced by development. Moreover, by employing a somewhat different form of autocollimator, limiting resolution can be reduced well below 10^{-3} arc-sec. Jones[4] of the University of Aberdeen, for example, has recorded angular resolution of 10^{-4} arc-sec with a small multislit autocollimator.

To get a better perspective of this excellent autocollimator resolution, it is necessary to consider the autocollimator's null stability. Null shift may be caused either by optomechanical or electro-optical factors. Optomechanical null shift is not analyzed readily, but it is undoubtedly minimized by constructing the pickoff of bonded quartz or of quartz and Invar. The one fundamental electro-optical source of null shift is the finite amount of light balanced out at null by the two detectors. This balanced light is a result of diffraction that causes the autocollimated prism nose image to be larger than the nose itself. In a nulled autocollimator utilizing two detectors per channel, a change in detector balance produces a residual signal that must be compensated by shifting the null point; a 10% change in detector balance will produce a 1-arc-sec null shift in the OPTAG I. The measured null shift is invariably smaller; in a two-week stability run, this unit exhibited a null shift of less than 0.2 arc-sec. Thus, it is null shift and not angular resolution that may actually limit autocollimator performance in many long-term applications. It should be noted that there are techniques whose use would permit a significant improvement in autocollimator stability, but there has been no indication of a need for a unit with much better than $\frac{1}{10}$-arc-sec stability.

To summarize, a miniature autocollimator, approximately 1 in. long by $\frac{3}{4}$ in. in diameter, has been tested and shown to be a desirable optical pickoff, responsive only to rotor angular deviation, with 1- to $\frac{1}{10}$-arc-sec over-all performance at present and capable of performance improvement beyond this.

Axial Autoreflector

Operating principles

The one known present application for the axial autoreflecting pickoff is on the gimbaled electrostatic gyro, which cannot tolerate distortion of its spherical rotor by the flat surface required by an autocollimating pickoff. In view of the symmetry of a spherical surface, rotor angular deviation can only be inferred from its surface translation. This may be sensed as the modulation in a reflected spot of light produced by a high-contrast pattern on the spinning surface. We will assume a center-fixed rotor to insure that surface translation is proportional to angular deviation. The two most obvious autoreflecting pickoff arrangements are those that observe either an equatorial or a polar pattern. Of the two, the axial pickoff that views a polar pattern is preferable both in theory and practice, and further discussion will be restricted to this form.

The axial pickoff (Fig. 4) is an autoreflector that projects a uniformly illuminated spot of light along the rotor spin axis and onto the rotor in the vicinity of a pole. Light reflected from the rotor is collected by the pickoff and transmitted via a beam splitter to an appropriate detector. The rotor pattern consists simply of high- and low-reflectivity areas whose boundary is a straight line passing more or less through the rotor pole. If the projected light spot is circular, it is easily shown that modulation of the reflected light occurs sinusoidally at spin frequency, with its amplitude proportional to rotor pole eccentricity within the spot, and its phase determined by the direction of eccentricity. Thus, there is no modulation when the pole is centered in the light spot. A simple reference pickoff (Fig. 5) somewhat removed from the pole is necessary for spatial resolution of the primary signal. The final axial autoreflector outputs, as for the autocollimator, indicate the direction and magnitude of rotor deviation about two orthogonal axes.

Because of its suspension characteristics, the electrostatic gyro may require translation sensors as part of a control system to maintain the rotor stably centered within its supporting electrode structure and/or to separate the effect of translation from the pickoff angle indication. From the sum and difference of two opposing axial autoreflector signals, both angular deviation and lateral translation may be measured. The addition of one or two equatorial pickoffs provides a measure of axial translation, although axial translation does not introduce a direct angular readout error as long as the axial pickoffs are well aligned and have sufficient depth of field. Thus, although the autoreflector is not inherently an angle sensor like the autocollimator, its char-

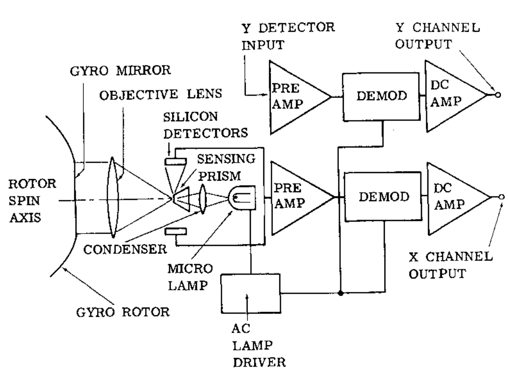

Fig. 3 Autocollimator functional block diagram.

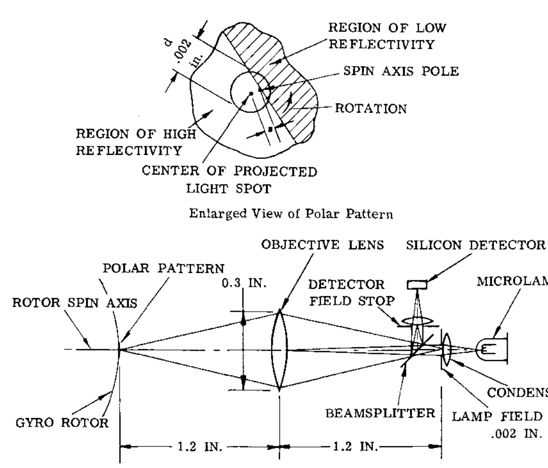

Fig. 4 Axial autoreflector (OPTAG II) schematic diagram.

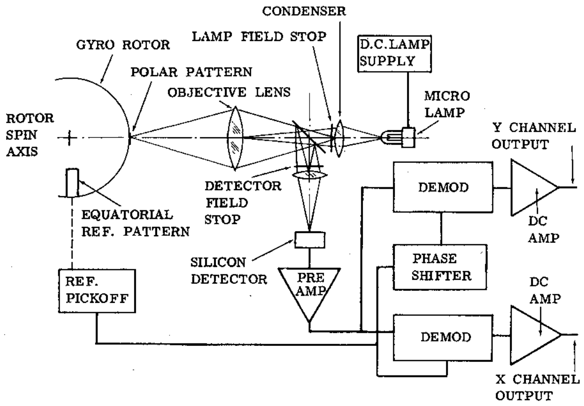

Fig. 5 Axial autoreflector functional block diagram.

acteristics are well matched to the needs of the gimbaled electrostatic gyro.

Performance evaluation

The axial autoreflecting pickoff of Fig. 4 can be investigated in a manner similar to the autocollimator to determine its theoretical translation resolution. The radiometric sensitivity is $P_S = NK_L w_0 A_S K_T K_\lambda K_v K_c \simeq 10^{-10}$ w/10-μin. spin axis eccentricity where

P_S = radiometric sensitivity, w/10-μin. spin axis eccentricity, within the 0.6- to 1.0-μ spectral region
N = radiance of W filament at 2150°K = 80 w/in.^{2}-sr
K_L = filament packing factor = 0.3
w_0 = solid angle of objective lens at sensing prism nose = 0.04 sr
A_s = swept area of changing reflectivity within light spot for 10-μin. decentering = ds = 2×10^{-8} in.^{2}
K_T = optical transmission efficiency (lamp-to-detector), including rotor and beam splitter losses = 0.04
K_λ = spectral utilization factor (lamp detector) = 0.1
K_v = over-all vignetting factor = 1 for spot size small with respect to rotor and short working distance
K_c = contrast factor given by $1 - R_P/R_R$, where R_R and R_P are, respectively, rotor and pattern reflectivities $\cong 0.8$.

The one unavoidable source of optical noise in an autoreflector is the power in the light spot, which has a magnitude P_0 = total power in spot image at detector $= A_0 P_S/A_s = (\pi d^2/8ds)\ 10^{-10} \cong 10^{-8}$ w. This is close to the diffracted power in the preceding autocollimator analysis, and the corresponding shot noise induced in a high quantum efficiency detector will have a magnitude approximating 10^{-13} w. The same conclusion follows, i.e., that resolution is presently limited by the larger detector noise. Again assuming a detector NEP of 10^{-11} w and a 1-cps bandwidth, we find that the autoreflector noise-equivalent-translation (NET) has a limiting value of NET $\equiv$ NEP/P_S = $10^{-11}/10^{-10}$ (10 μin.) = 1 μin. This corresponds to approximately $\frac{1}{5}$ arc-sec on a 2-in.-diam rotor. In practice, the OPTAG IIA (Fig. 6) has exhibited translational resolution of 10 μin., although accuracy may have been limited by the test equipment employed, some of which must be of exceedingly precise construction.

As in the case of the autocollimator, the autoreflector design is not optimum from a theoretical standpoint but has been dictated by practical considerations. As indicated in the preceding analysis, photocurrent shot noise is well below the assumed detector NEP; consequently, the pickoff spot diameter could usefully be increased up to 100 times, providing shot-noise-limited resolution of 10^{-2} μin.in theory, but a large, high-power lamp would be required to do so. Other noise sources and null stability considerations also set a practical (rather than analytical) limit on the practically achievable and useful resolution: nonuniform spot illumination and noncircular spot geometry, each of which can cause noise at the spin frequency of much greater magnitude than the theoretical angular resolution. On the other hand, null shift, caused either by uncompensated rotor translation, lamp-filament movement, or by minute pickoff shift on the gyro case, is more likely to be the most important limiting factor. Thus, although autoreflector performance can theoretically reach much lower, it is restricted in practice to the 1- to 10-μin. region.

Autocollimator-Autoreflector Comparison

The autocollimator is a more desirable angle pickoff than the autoreflector, principally because of its true angle-sensing capability that leaves its performance unaffected by extraneous rotor translation. In addition, its inherently superior noise characteristics will permit greater performance improvement as better detectors, lamps, and other components are developed. Another advantage is that the autocollimator observes a flat mirror whose attitude change is detected only as a low-frequency or d.c. image shift, whereas the axial autoreflector detects rotor surface translation as an amplitude variation of the spin frequency component of the energy in a fixed image. Thus, autoreflector operation is wholly dependent on the external modulation produced by rotor spin. The fact that the autocollimator can operate with a nonrotating mirror makes it far easier to test. This characteristic can also permit the testing of a gyro with its rotor stationary; in this way, residual suspension torques may be detected more easily because they are not resisted by the high-angular momentum of a spinning rotor.

Direct-Readout Pickoffs

Description and Operating Principles

Because its rotor attitude must be measured in any orientation, the readout system is one of the strap-down gyro's most severe development problems. The gimbaled gyro requires only one null-sensing pickoff to sense rotor deviation from null alignment, whereas rotor attitude is measured by rotary transducers on the gimbal axes. In contrast to this, the strap-down gyro requires a set of pickoffs, usually three orthogonal units, together with a digital computer to determine rotor attitude. This increased strap-down pickoff system complexity is not unexpected, however, for the strap-down gyro represents the first departure from the classical gimbaled gyro with electromechanical readout, and some of the complexity of the eliminated gimbal mechanism must reappear in another form.

The strap-down gyro readout system is required to measure the attitude of the spin axis of a spinning spherical rotor

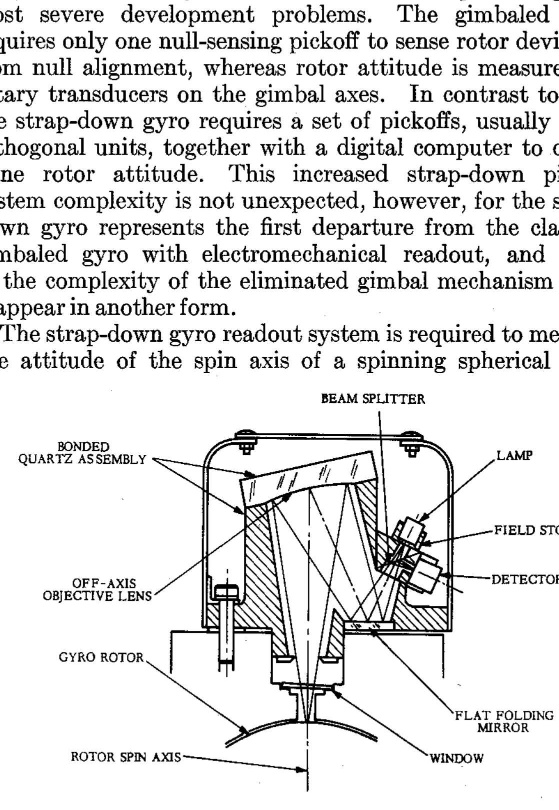

Fig. 6 OPTAG IIA internal arrangement. (This design has been superseded by more advanced forms.)

with respect to, say, a Cartesian reference frame (Fig. 7). Assume that three autoreflecting, transit-sensing pickoffs are located along the three orthogonal reference axes. Each pickoff scans the rotor with a small light spot along a latitude circle. With the rotor spinning at constant speed it is possible, by measurement of the reflected light modulation produced by an appropriate surface pattern, to establish the latitude of each pickoff and, from this information, to compute spin axis attitude.

One possible pattern is a pair of lines originating at the poles, separated at each latitude so that the ratio of the short and long arcs has the magnitude of the cosine of colatitude. This pattern gives the direction cosines of the spin axis vector. Various other patterns can be employed; those using great circles are preferable because they can be applied most easily and accurately, but attitude computation is more complex than when the pickoffs directly indicate trigonometric functions of latitude.

The actual pickoff readout is based on time measurement. The only known working systems use line patterns, and their latitude circle arc lengths are indicated by the number of high-frequency clock pulses transmitted in the time interval between line transits through the pickoff spot. These systems generate the desired function of latitude by taking the ratio of the short and long latitude circle arcs, so that readout system accuracy is left unaffected by changes in rotor speed. Because the orthogonal pickoff set tends to become inaccurate as any one pickoff approaches a pole, it may be necessary to employ redundant pickoffs or an added pattern to provide a continuously accurate indication of rotor attitude.

Performance Evaluation

In a strap-down guidance system, two gyros are required for three-axis attitude determination. Gyro attitude information is needed both for steering and for resolution of accelerometer signals. The accelerometers may have random attitudes with respect to gravity, and the large gravity acceleration component must be very well defined if the motional acceleration component is to be gaged accurately. This imposes a rather stringent requirement on gyro drift, and its magnitude cannot exceed the leveling tolerance required of more conventional guidance systems, i.e., 20 arc-sec or less. This implies that the pickoff system must be accurate to 10 arc-sec or better.

The formidable strap-down readout problem is illustrated by the following example of transit autoreflector performance. The transit autoreflector has the same optical form as the axial autoreflector, and the specific unit to be evaluated is that of Fig. 4. Consider a 2-in.-diam rotor spinning at 12,000 rpm observed at a 45° polar latitude by an autoreflecting pickoff. The rotor has a 2-line pattern originating at a rotor pole, the ratio of whose short and long arcs along any latitude circle has the magnitude of the cosine of the colatitude. In a 10-arc-sec latitude change at 45°, the cosine magnitude and, hence, the arc ratio, changes by

$$\Delta \cos\lambda_{45°} = \sin\lambda\Delta\lambda = 0.7 \times 5 \times 10^{-5} = 3.5 \times 10^{-5}$$

where λ = colatitude. Since the latitude-circle circumference is about $S_\lambda = 2\pi \sin\lambda = 4$ in., the line-pattern accuracy must be considerably better than $\frac{1}{2}S_\lambda\Delta\cos\lambda = 7 \times 10^{-5}$ in.

It is now assumed that for an information bandwidth of 1 cps the individual cosine measurements occuring at a rate of 200/sec are averaged over 1 sec. This is equivalent to saying that the effective noise bandwidth, regardless of the actual rotor velocity, is the bandwidth required to sense line transit with the necessary accuracy if the rotor velocity is 1 rps. In this case, the effective integrating time for detecting line transit must be considerably less than $\Delta t = \frac{1}{2}\Delta \cos\lambda = 1.7 \times 10^{-5}$ sec, and to observe a single transit within this

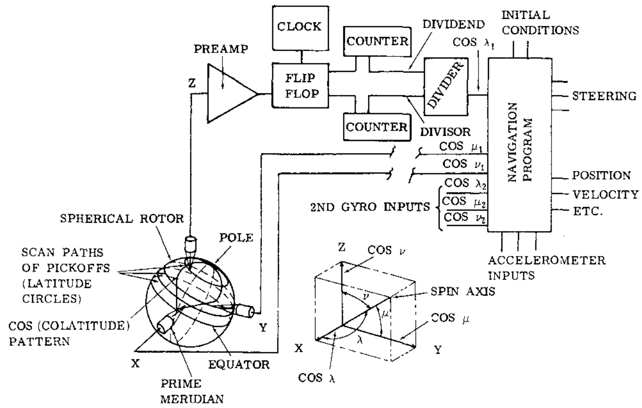

Fig. 7 Strap-down gyro readout system functional block diagram.

time requires a bandwidth of about 15 kc. (The actual transit at 200 rps must be detected in 10^{-7} sec, corresponding to a signal bandwidth of about 3 Mc.) If the autoreflector spot is to be traversed in this time, it must be of the order of $7 \times 10^{-5} \cong 10^{-4}$-in. diam, and its power content is $P_{0_{10^{-4}}} = 3 \times 10^{-8} (10^{-4}\ 12 \times 10^{-3})^2 \cong 10^{-10}$ w. To achieve a signal-to-noise ratio of, say 4, at maximum signal requires that the detector NEP be no higher than $P_{N_{max}} = 10^{-10}/4(1.5 \times 10^4)^{1/2} = 2 \times 10^{-13}$ w.

No solid-state detector presently possesses so low an NEP, and thus, a photomultiplier detector is required to achieve 10-arc-sec resolution under the assumed conditions. Some reduction in detector quality might be permitted if the autoreflector spot size is increased. The result of this change is to raise the maximum signal level, but it is accompanied by increased noise-in-signal. In addition, the increase in signal rise time requires more critical control of the readout counter triggering level, as well as far better over-all system gain stability.

Assuming a solid-state detector NEP of 10^{-11} w, the optical signal level required for a signal-to-noise ratio of 4 with a noise bandwidth of 15 kc is $P_S = 4 \times 10^{-11} \times (1.5 \times 10^4)^{1/2} = 5 \times 10^{-9}$ w. This signal level is not that given by the whole spot, however, once it becomes larger than 10^{-4} in., because the requirement is to sense line transit through a centered region 10^{-4} in. wide. To achieve this signal level in that region requires a spot about

$$[\pi(10^{-3})^2/(3 \times 10^{-8})][(5 \times 10^{-9})/10^{-4}] = 0.005 \text{ in. diam}$$

The average shot-noise level from this spot size in a 15 kc bandwidth is $10^{-13} \times 0.005/0.002\ [(1.5 \times 10^4)/2]^{1/2} = 1.5 \times 10^{-11}$ w, which is approximately equal to detector NEP.

Thus, it is seen that the detector and electronic problems for a strap-down gyro attitude-measuring system are solvable possibly at the expense of a somewhat bulky package if photomultipliers are used. The most difficult problem, however, is to apply with sufficient accuracy a rotor pattern from which gyro attitude may be derived with a minimum of signal processing electronics. The difficulty stems from the 10^{-5}-in.-pattern accuracy and from the curved form of the desirable patterns. Although great circle patterns are relatively easy to produce, they require complex signal processing. As far as pickoff performance is concerned, early OPTAG III transit-sensing pickoffs using a solid-state detector exhibited triggering repeatability of about 1μ sec. Newer designs are expected to improve on this performance significantly.

Miscellaneous Considerations

Pickoff Size

An interesting aspect of pickoff design is the effect of size change on performance. Assuming that an optimized autocollimating pickoff can be scaled to any size so as to maintain all angular relations, this requires that the lens focal length, detector size, and the prism nose dimension must change directly with the lens diameter. (The larger flat mirror area required by a larger pickoff implies, for consistency, that the rotor eventually must grow accordingly.) Angular sensitivity is proportional to the product of the nose dimension and the lens focal length and, therefore, increases with the (lens diameter)^2, or $P_\theta \propto d^2$. Detector-noise increases with the detector dimension and therefore with the lens diameter, or $P_{N_D} \propto d$. Shot-noise varies with the square root of the product of the diffraction pattern angular area (that remains constant) and the sensitivity and, therefore, increases directly with lens diameter or $P_{N_S} \propto (d \times d^{-1} \times d^2)^{1/2} = d$.

When a pickoff is detector-noise-limited, the angular resolution given by the ratio of detector-noise ($\propto d$) to sensitivity ($\propto d^2$) reduces as (lens diameter)$^{-1}$, or NEA $= (P_{N_D}/P_0) \propto d/d^2 = d^{-1}$. The performance improvement obtained by increasing the pickoff size is somewhat costly, however, for the pickoff volume and weight increase as (NEA)$^{-3}$. A second distinct case is encountered when the pickoff is shot-noise-limited. The resolution in this case is given by the ratio of shot-noise ($\propto d$) to sensitivity, and, therefore, here also NEA $\propto d^{-1}$, and the volume and weight vary with (NEA)$^{-3}$. Thus, in an optimized autocollimating pickoff, regardless of whether it is detector- or shot-noise-limited, angular resolution can be reduced by increasing the lens diameter, but the pickoff volume and weight increase at a much faster rate. The resolution per size per weight tradeoff, indeed, favors much smaller pickoffs than the OPTAG I and II, and these units were made as large as they are principally because of difficulty anticipated in making a sufficiently precise and stable assembly of much smaller parts.

Although the change in angular resolution with size is the same in either the detector- or shot-noise-limited case, the latter condition is the more fundamental limitation and provides the best possible performance. In the autocollimating OPTAG I, for example, the computed angular resolution would improve from 10^{-3} arc-sec to about 5×10^{-5} arc-sec if the detector-noise level was reduced from the assumed 10^{-11} w to below the 5×10^{-13}-w shot-noise level. Inasmuch as the performance improvement attainable from better detectors is so much greater than that permitted by increasing pickoff size, it is unquestionably more rewarding to employ the former approach to performance improvement with any pickoff that is not shot-noise-limited. Resolution may also be improved by increasing the source radiance and/or the prism nose angular subtense. Thus, it is difficult to establish criteria for the forementioned "optimized" design, and, in a real situation, performance improvement may be obtained by a number of effective and desirable alternatives to increasing pickoff size.

To vary the scale of an autoreflector in a manner consistent with the autocollimator all angular relations again must be maintained, and, thus, the size of the illuminated spot changes with the lens diameter. Pickoff performance is ultimately specified in angular terms, but the pickoff actually senses rotor spin axis eccentricity, and, therefore, the angular performance is determined by rotor size. Consequently, the case of both fixed- and variable-size rotors must be considered.

Autoreflector sensitivity to spin axis eccentricity varies directly with its lens diameter as do both detector-noise and shot-noise. Thus, an optimized autoreflector exhibits constant resolution in spin axis eccentricity in either the detector- or shot-noise-limited mode independent of its size. Furthermore, in the shot-noise-limited mode, the resolution is independent of spot size. If the rotor size is fixed, the pickoff angular resolution is correspondingly constant. On the other hand, if the rotor and pickoff size change together, angular resolution improves with d^{-1} as for the autocollimator. Thus, for a given gyro whose size is fixed, an optimized shot-noise-limited pickoff is most desirably made as small as possible inasmuch as its angular resolution is unaffected by its size. Here again, however, in a real situation improvement may be possible by the use of a better or smaller detector or a brighter lamp that permits the transition from detector- to shot-noise-limited resolution.

Angular Range and Linearity

Generally, only a small angular range and relatively mediocre linearity are required of null-sensing pickoffs, although output symmetry should be quite good to avoid development of an offset bias in the presence of oscillation about null. The autocollimator is inherently linear in either one of its two channels. Its angular range is equal to $\pm\frac{1}{2}$ its prism nose angular subtense, or somewhat more than $\pm\frac{1}{2}°$ for the OPTAG I. Linearity and symmetry are of the order of 1% over a range of several arc minutes. There are no fundamental restrictions on these characteristics, and they may be improved within reasonable limits by careful alignment and balance. The biaxial autocollimator does suffer from gain-change and nonlinearity in one channel in the presence of a significant signal in the other channel. This effect is usually negligible under normal operating conditions where the error angles are very small with respect to the prism nose angular subtense.

The one other angular characteristic of interest is angular acquisition range. This is the angular range about null within which a useful signal is detected by the pickoff and must necessarily be larger than the initial rotor misalignment to insure that the gimbal servos are controlled to drive the pickoff toward the null point. Acquisition range is determined by the dimensions of both the prism sides and the detectors, which must be large enough to permit a part of the nose image to reach the detector over the desired range. Thus, for most purposes, the size of the illuminated prism nose is chosen to obtain good sensitivity, which increases with the nose dimension, and to keep gain-change and nonlinearity effects within tolerance, whereas the detector and prism size are selected to achieve a given acquisition range.

The mechanism of signal generation in the axial autoreflecting pickoff involves a patterned rotor that rotates about a spin axis that may fall anywhere in or out of a small illuminated spot (Fig. 4). When the spin axis lies within $\frac{1}{3}$ radius of the spot center, the error signal magnitude varies linearly with eccentricity within a few percent. By the time the spin axis reaches the spot periphery, the error signal has almost saturated and maintains approximately the same magnitude as the spin axis moves large distances outside the spot. For a 2-in.-diam rotor and a 0.002-in.-diam spot, the autoreflector has a linear angular range of only about $\frac{1}{3}$ mil or about 1 min of arc. The error signal continues to grow, but at a decreasing rate, up to 1 mil or $3\frac{1}{3}$ arc-min at the edge of the spot, beyond which the saturated signal prevails. Thus, the autoreflector has a highly nonlinear transfer characteristic except for a very small region near null. The saturated signal outside the spot serves for acquistion purposes over an extremely wide angular range. Unlike the autocollimator, the axial autoreflector acquisition range is independent of pickoff geometry, and an acquisition signal is available up to angles of 20° or more from null.

Null Stability

Optomechanical null stability of an optical gyro pickoff is determined principally by its optomechanical structure that is basically nonanalytic, i.e., a satisfactory mathematical model of the behavior of materials over long periods of time has not yet been devised. Only the upper limit of autocollimator electro-optical stability can be established mathematically, and this was done previously, where it was observed that the primary electro-optical source of null shift is the two detectors in each channel that presumably can become unbalanced, causing a shift of as much as 1 arc-sec for a 10% unbalance.

At the cost of added complexity and moving parts, it is possible to construct a small autocollimator using only one detector per channel and with precisely uniform prism nose illumination, thereby reducing probable electro-optical null shift to the $\frac{1}{100}$ arc-sec level. Once this is accomplished, the only other possible source of null shift is the relative movement of the optomechanical structure. As previously suggested, this movement with time and temperature can be minimized by the use of a bonded structure of fused quartz, the solid exhibiting the lowest expansivity and best stability of all engineering materials. An all-reflective (Fig. 6) or catadioptric (combination of reflective and refractive elements) lens design permits the use of fused quartz throughout the pickoff. A pickoff of this construction, with electro-optical null shift minimized, would be more stable than the gyro with which it might be used. This stability can be further enhanced by containing the pickoff in a temperature-controlled environment. There is little doubt that a well-made pickoff constructed along these lines and with thermal control would be stable to $\frac{1}{100}$ arc-sec over periods of weeks or months.

Environmental Resistance

Because of the small size, light weight, and stationary nature of all parts of the gyro pickoffs, they can easily be made to withstand a 100–200*g* motional environment without failure or significant null shift. The one possible exception is the pickoff light source, presently an incandescent lamp. These lamps have been operated at 200*g* in a centrifuge without failure and presumably would withstand any motional environment the gyro is capable of surviving, but lamp-filament movement could nevertheless cause at least a temporary null shift, particularly in the autoreflecting pickoffs. For a greater margin of resistance in the future, either gallium arsenide recombination-emission diodes or neon glow lamps might be used. The latter are far more rugged than filament lamps; their obvious drawback is low ouput that leaves the pickoff highly susceptible to pickup and necessitates extremely high-quality, low-noise, high-gain electronics. Early Ga-As diodes were tried in 1963 in an attempt to make the first all solid-state autocollimator, but the tests were discontinued because the room-temperature units provided insufficient signal and were extremely temperature-sensitive. Nevertheless, it is expected that gyro optical pickoffs will shortly be of completely solid-state construction and capable of withstanding any foreseeable shock and vibration.

The predictable effects of temperature change are dimensional changes in all parts of the pickoff and a variation in the detector-noise level due to the change in the number of thermally stimulated electrons. Neither of these changes is inherently a source of null shift, and, in fact, detector-noise regardless of its level can only increase pickoff NEA but cannot cause null shift as long as differential gain change is avoided by appropriately loading the detectors.

Asymmetric dimensional change can cause null shift and may result from temperature asymmetry, expansivity variation, or an asymmetric form. Temperature asymmetry may be minimized by appropriate shrouding to provide a uniform thermal environment around the pickoff and by the use of high-conductivity materials. Expansivity variation is obviously reduced on an absolute basis by using materials with the lowest expansivity. Interestingly, it is found that most engineering materials have almost the same ratio of expansivity to conductivity that would make all equally good in resisting thermal transients. In general, however, the low expansivity materials are preferable (fused quartz, in particular, because of its stability). An asymmetric pickoff form encourages temperature gradients and, moreover, may cause null shift from a uniform temperature change.

There has been little temperature testing of the pickoffs, partly because of the difficulty of isolating pickoff changes from those of the test apparatus. The autoreflecting pickoff is particularly difficult to temperature test in a valid way because of its necessary proximity and direct mechanical coupling to a test rotor assembly that cannot avoid being extremely temperature-sensitive.

The most trustworthy temperature tests results were inadvertantly obtained in the OPTAG I stability test, wherein the pickoff was seated directly on a quartz test mirror. It was found that the primary null shift observed was a diurnal variation in synchronism with the test area daily temperature cycle and with a scale factor of about $\frac{1}{10}$ arc-sec/°C. A comprehensive investigation of this temperature sensitivity was not attempted, but it is quite certain that much better temperature stability can be obtained easily by minor design revision and changes in assembly technique of the symmetrical, low expansivity OPTAG I. The asymmetric OPTAG II is predictably more fundamentally temperature-sensitive and redesign to a symmetrical form would be required to exploit more fully its all-quartz construction. As a practical matter the gyros with which the pickoffs are used are undoubtedly far more temperature-sensitive than the pickoffs, and their need for precise temperature control will provide an ideal thermal environment for the pickoffs.

With respect to ionizing radiation, the pickoffs are inert except for their detectors and their electronics. The detectors are very similar to silicon and other photovoltaic solar cells that have been developed to an excellent state of radiation resistance because of the need for long-lived satellite and space-vehicle power supplies. Moreover, silicon and other types of solid-state detectors have exhibited long useful lifetimes in satellites. Similarly, standard electronic circuits of the type required in pickoffs have operated for years in space. Thus it is safe to assume that optical pickoffs can be made sufficiently radiation-resistant to last as long as any other components subjected to ionizing radiation in space.

The only element in the optical pickoffs described that would have a predicted lifetime of less than 100,000 hr is the light source. Fortunately, tungsten-filament lamp life shows an extremely large variation with filament temperature.[3] Over the 2000–3000°K temperature range, the life ratio varies approximately as the 35th power of the inverse temperature ratio. On the other hand, the total radiation from 0.6–1 μ (in the Si spectral region) only varies approximately as the 8th power of the temperature ratio. Thus, filament life increases approximately as the inverse 4th power of effective power reduction, permitting a relatively desirable tradeoff between sensitivity and lifetime. It is assumed in the analyses that the pickoff lamp operates at a temperature of 2150°K. A lamp using a 1.5-mil-diam filament at this temperature will have a reliable lifetime of well over 50,000 hr, which is sufficient for most applications. By reducing the temperature below 2000°K, lamp lifetime can be extended to above 2×10^6 hr (over 200 yr). Therefore, if the pickoff is well constructed, the intrinsic reliability of all components can be expected to provide a mean-time-to-failure of from 25,000 to 100,000 hr.

It should be noted that the incandescent lamp is a relatively efficient radiation source for devices using detectors sensitive in the near infrared. At 2150°K, 10% of a lamp's output falls below 1 μ and 55% below 2 μ, whereas at 2400°K, 15% falls below 1 μ and 62% below 2 μ. Since essentially all the lamp input power is converted to short wavelength radiation, these output numbers can be considered as efficiency levels. Compared to these magnitudes, the $\frac{1}{10}$% or lower efficiency of a Ga-As lamp is exceedingly poor, whereas glow lamps are not much better.

Power Requirements

The only significant power required above the low mw level is that for the lamp. A typical lamp may require 250 ma at 3 v, although lamps of $\frac{1}{10}$ w or less are available and can be employed at some sacrifice in sensitivity if power drain must be minimized. Although the pickoff electronics are well adapted to 28-v d.c. operation, it is desirable to have a.c. available both to permit convenient transformation down to the relatively low lamp voltages and to provide the modulation required by the autocollimating pickoff. Alternatively, a solid-state converter may be used for modulation and voltage transformation. In the case where lamp modulation is necessary, a converter that applies narrow 28-v pulses to the lamp has been found to provide excellent operation with a minimum of parts. In any case, the optical pickoffs may be considered to require no more than 1 w, and this may be reduced to $\frac{1}{10}$ w if necessary.

Conclusions

It has been shown that optical gyro pickoffs are available with accuracy to the order of $\frac{1}{10}$ arc-sec and that considerable performance improvement is possible. Modest improvements can be achieved by optimizing present pickoffs, whereas large improvements are dependent upon the development of better pickoff components.

It is of interest to consider how good pickoffs may have to be if they are to keep pace with the probable increase in gyro performance. Newton[5] has assumed that thermal fluctuations will prove to be the ultimate limitation in gyro performance, as they are for so many electrical and mechanical devices. On this basis he shows that present gyros are several magnitudes removed from perfection. In particular, he quotes a time-dependent deviation factor of about 10^{-4} arc-sec/sec$^{1/2}$ for a typical thermal-noise-limited gyro. If his assumptions are correct, then those developing both gyros and pickoffs must accept the challenge of overcoming the large performance gap that presently exists between theory and practice.

References

[1] Slater, J. M., "Exotic gyros," Control Eng. **9**, 92–97 (November 1962).

[2] Jones, H. A. and Langmuir, I. "Characteristics of tungsten filaments as functions of temperature," Gen. Elec. Rev. **30**, 310–319, 354–361, 409–412 (1927).

[3] Williams, R. L., "Fast high-sensitivity silicon photodiodes," J. Opt. Soc. Am. **52**, 1237–1244 (1962).

[4] Jones, R. V., "Some developments and applications of the optical lever," J. Sci. Instr. **38**, 37–45 (February 1961).

[5] Newton, G. C., "Inertial-guidance limitations imposed by fluctuation phenomena in gyroscopes," Proc. Inst. Radio Engrs. **48**, 520–527 (1960).